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Review Article

Mathematics problem-solving research in high school education: Trends and insights from the Scopus database (1983–2023)

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ABSTRACT

Problem-solving competency is crucial for social development, especially in complex environments. In mathematics education, problem-solving enhances logic, creativity, and analytical skills, contributing to societal progress. This article identified quantitative information about important publications, authors, resources, and research trends on mathematics problem-solving in high school education using the bibliometric analysis method. The input data is a set of 334 publications from the Scopus database published over four decades from 1983 to 2023. The results show that this field has obtained increasing interest, particularly in the last five years, with the USA and Indonesia being the countries with the most publications and Santos-Trigo and Putri Rii being the most influential authors. Three research trends include problem-solving in teaching mathematics in high schools, especially in teaching geometry and algebra; developing problem-solving and computational thinking skills through STEM education, engineering education, and educational computing for students; and using information technology to solve mathematics problems. These results provide teachers and researchers with helpful information about solving mathematical problems in general education, thereby contributing to shaping and proposing effective research and educational strategies, new teaching methods, training programs, and appropriate educational policies.

Keywords: problem-solving, mathematics problems, high school, bibliometric, Scopus

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INTRODUCTION

Learning in the 21st century requires students to have problem-solving, critical thinking, collaboration, creativity, and communication competencies (Pramasdyahsari et al., 2023). Students skilled in a deep understanding of problem-solving can find original solutions (Putri et al., 2023). But the current problem is that students still have limited and inadequate creativity or creative and innovative problem-solving skills (Yusuf et al., 2023).

In high school, mathematics, a subject with a high generality and unique characteristics of mathematical science, has much potential to foster problem-solving competency (Yanase & Fujioka, 2007). On the other hand, in teaching mathematics, specifically teaching concepts, teaching theorems, and teaching solving mathematics exercises, each has its important role and unique characteristics that contribute to developing problem-solving competency. Besides, mathematics is present everywhere: in offices, agencies, organizations, companies, factories, enterprises, science (literature, history, geography, physics, etc.), sports, and arts (sculpture, painting, music, etc.). And so, using mathematical concepts and tools are everyday activities that everyone does, not only including students or researchers having to use mathematics, but every ordinary person having to use, with a fundamental understanding of mathematics and mathematics-related capacities such as computing capacity, modelling capacity, logical thinking capacity, problem-solving capacity, etc.

There have been a number of studies conducted in similar directions such as general studies on problem-based learning (PBL), in which research on PBL is conducted by a bibliometric analysis (de Pinho et al., 2015; Hallinger, 2021), or research trends in PBL in middle school (Samosir et al., 2023). There has also been research on mathematics problem-solving in elementary education through a bibliometric analysis (Suseelan et al., 2022).

Through this article, the authors aim to have a comprehensive overview of the development of global research related to mathematics problem-solving in high schools (including mathematics problem-solving and problem-solving in life or other subjects with mathematics used as a tool), thereby providing useful information for scholars to conduct their future research in this direction. In this study, the researchers focus on answering the following research questions:

(1) the annual growth rate of the number of publications related to mathematics problem-solving in high school and its citations,

(2) the most productive countries and the scholars with the most publications and citations in their collections and collaborations,

(3) the most popular publication journals in mathematics problem-solving in high schools,

(4) the articles with the most citations, and

(5) the most important research topics in the topic.

METHOD

There are several different approaches to answering the above research questions. Among them, bibliometric analysis is essential for quantitatively investigating scientific activities, offering a structured approach to uncover trends and contributions in a specific field. This method provides valuable insights into the development of scientific knowledge by analyzing the metadata of research publications (Zupic & Čater, 2015). In the context of this study, bibliometric analysis is particularly well-suited because it systematically maps the intellectual landscape of mathematics problem-solving research in high school education, enabling the identification of pivotal research trends, influential publications, and key contributors. Data for this study are collected from the Scopus database, the largest and most widely used scientific database available (Hallinger & Nguyen, 2020).

Based on the analysis in the Introduction, the concept of 'problem-solving' is construed in two aspects including: The use of 'problem-solving' in mathematics and the use of mathematics as a 'problem-solving' tool in other subjects or integrated content. For the selection of search keywords, we refer to the previous research on the same topic by Suseelan et al. (2022), accordingly, the keywords include: 'problem-solving', 'word problems', or 'mathematics problems'. The authors added the keywords 'math*' and 'high school' to match the research topic. The OR and AND operators combine the keywords accordingly. Such keywords are searched in the document's abstract, keywords, and title. The data is limited to the following parameters: document types are article, conference paper, book chapter, and review; subject areas are social sciences or math; language is English; and published time before 2024. The query statements from the Scopus database are, as follows: TITLE-ABS-KEY (("problem solving" OR "problem-solving" OR "word problems" OR "mathematics problems") AND math* AND "high school") AND PUBYEAR < 2024 AND (LIMIT-TO (SUBJAREA, "soci") OR LIMIT-TO (SUBJAREA, "math")) AND (LIMIT-TO (DOCTYPE, "ar") OR LIMIT-TO (DOCTYPE, "cp") OR LIMIT-TO (DOCTYPE, "ch") OR LIMIT-TO (DOCTYPE, "re")). The data query results on December 27, 2023, return 611 documents.

Information regarding the documents' titles, abstracts, and keywords was meticulously extracted for review by two independent authors. These authors evaluated and identified documents unrelated to the research topic. A consensus was reached to ensure accuracy by comparing the list of excluded documents between the two reviewers. The primary reasons for exclusion were, as follows:

(1) the search keywords appeared only in the research context section of the abstract and

(2) the research problem was not directly addressed in the document content.

Following this rigorous filtering process, 227 documents were excluded, resulting in a final dataset of 334 high-quality publications. This refined dataset served as the basis for initial analyses using tools provided by Scopus to gather additional metadata, such as information on authors, affiliations, and journals. The metadata of the final publication collection was subsequently exported in CSV format for further processing using bibliometric analysis tools. Supplementary data from the Scimago Journal & Country Rank (https://www.scimagojr.com/) was also incorporated to enhance the robustness of the analysis.

This study employed Biblioshiny (version 4.2.0) and VOSviewer (version 1.6.20) to conduct bibliometric analyses. Biblioshiny, an open-source tool, enables comprehensive cartographic analysis of scientific literature, offering robust visualization capabilities for identifying patterns, trends, and key themes in research (Aria & Cuccurullo, 2017). Meanwhile, VOSviewer is designed to construct and visualize bibliometric networks, including co-authorship, citation, and keyword co-occurrence networks. Its advanced text-mining capabilities facilitate extracting and mapping significant terms from scientific literature, enabling the development of detailed co-occurrence networks (van Eck & Waltman, 2010). These tools are essential for this study as they complement each other in achieving a nuanced understanding of the research landscape. Biblioshiny's thematic mapping capabilities allow researchers to track evolving research areas and identify core and emerging topics, which is critical for understanding trends in mathematics problem-solving research. VOSviewer's ability to generate network visualizations highlights the interconnections between key concepts, authors, and publications, offering insights into the intellectual structure of the field. Together, these tools enable systematic and in-depth dataset exploration, ensuring the study's findings are comprehensive and robust. Their combined application provides a methodological rigor that traditional analysis methods alone cannot achieve, justifying their necessity in this research.

RESULTS AND DISCUSSION

Publication Output and Growth Trend

The main information about the collected data set is shown in **Table 1**. During the period of 1983–2023, a total of 334 documents are extracted from the Scopus database, including 242 articles (accounting for 72.46%), 13 book chapters (accounting for 3.89%) and 79 conference papers (accounting for 23.65%). The document average age is 9.72. These documents are published in 187 sources with an average annual growth rate of 7.49%. The total number of participating authors is 852, of which 56 authors have published only 1 article. The average collaboration rate per document is 2.83, of which the international collaboration rate is 7.485%. h-index of this collection is 32 with a total number of 5185 citations and the average citations of each document that is 15.52. In addition, there are a total of 904 keywords and 10,771 cited documents in the above publications.

Figure 1 shows the number of publications published yearly and the cumulative number of citations of related research on mathematics problem-solving in high school.

Table 1. Main information collected about mathematics problem-solving in high school

Description	Results
Main information about data	
Timespan	1983:2023
Sources (journals, books, etc.)	187
Documents	334
Annual growth rate (%)	7.490
Document average age	9.720
Average citations per document	15.520
References	10,771
Document contents	
Keywords plus (ID)	753
Author's keywords (DE)	904
Authors	
Authors	852
Authors of single-authored documents	56
Authors collaboration	
Single-authored documents	62
Co-authors per document	2.830
International co-authorships (%)	7.485
Document types	
Article	242
Book chapter	13
Conference paper	79

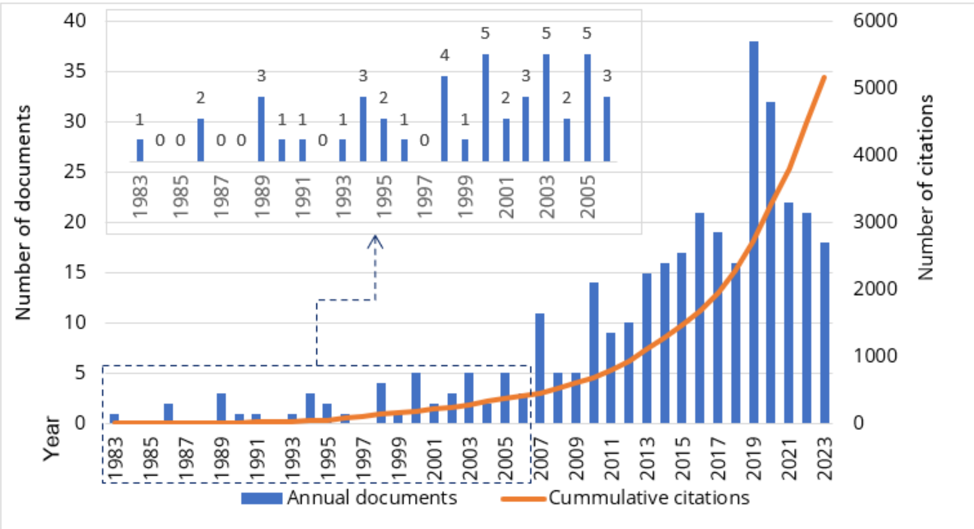

Figure 1. Increase in the number of publications on mathematics problem-solving in high school and cumulative citations for those publications (Source: Authors' own elaboration)

However, there has been an uneven growth rate over the years. The first article on related research on mathematics problem-solving in high school is "knowledge sources for an intelligent algebra tutor" published in 1986 in Computational Intelligence Magazine (Bregar et al., 1986). This article focuses on providing a fundamental knowledge base for an intelligent guidance system for algebra problems in high school. This model is based on the analysis of protocols between students and experts/tutors, allowing simulation of many different problem-solving behaviors and being easily extended to other problems in new fields as possible, allowing students' errors to be monitored and remedied, as well as providing an approach to understanding problem difficulty so that challenging problems can be created, and plans are prepared to solve them.

From 1983 to 2006, mathematics problem-solving in high school did not obtain much attention from scientists, and research on this topic only appeared in very few studies. From 2007 to 2023, the number of studies obtained an unstable growth, while from 2011–2016, witnessed a steady growth, particularly the number of publications on this issue had a breakthrough increase in 2019 and 2020. Notably, in 2019, there were 38 publications; in 2020, 32 were published. However, in the following years, the number of publications

Table 2. Top 8 most influential countries

Rank	Country	TP	%	TC	%	TC/TP	SCP	MCP
1	USA	67	33.0	2,989	73.7	44.6	64	3
2	Indonesia	24	11.8	173	4.3	7.2	22	2
3	China	14	6.9	152	3.7	10.9	13	1
4	Israel	7	3.4	45	1.1	6.4	7	0
5	Mexico	7	3.4	55	1.4	7.9	6	1
6	Turkey	7	3.4	46	1.1	6.6	7	0
7	Brazil	6	3.0	29	0.7	4.8	5	1
8	Germany	5	2.5	85	2.1	17.0	5	0

Note. TP: Total publications; TC: Total citation; SCP: Single country publications (author collaboration with single country publication); MCP: Multiple country publication (author collaboration with multiple country publication).

gradually decreased. The reason for this decline is that research on mathematics problem-solving has been expanded to many other fields such as artificial intelligence and machine learning, optimization, computer science or other fields of economy, in artificial intelligence and machine learning, designing algorithms and machine learning models to solve complex mathematical problems plays an important role, for example deep learning-based intelligent assistant systems for mathematics education (Lample & Charton, 2020), in solving function optimization problems, linear and non-linear programming techniques, real-world mathematics problem-solving applied to find optimal solutions (Hillier & Lieberman, 2018), in computer science, using algorithms and effective data structures for mathematics problem-solving is an indispensable part, for example solving systems of linear equations, finding the shortest path, etc. (Radhakrishnan et al., 2007), in finance, investment, macroeconomics and microeconomics, problem-solving is done based on mathematical modelling, game theory and probability (Samuelson & Nordhaus, 2009). So, the keyword PBL is absent, though the nature of mathematics problem-solving is still implicit in the research.

Contribution by Nations

According to data retrieved from Scopus, researchers from 36 countries have contributed to studies on this topic. A publication is attributed to a country if at least one author or co-author is affiliated with an institution located in that country. **Table 2** highlights the top 8 countries with the highest publications on mathematics problem-solving in high school education. The USA leads with 67 publications (including 3 MCP articles and 64 SCP articles), accounting for 33.0% of the total publications. The number of citations in the USA is 2989, accounting for the highest rate of 73.7% of the total number of citations. The citation/publication ratio in the USA is 44.6. Indonesia ranks second with 24 publications (including 2 MCP articles and 22 SCP articles), accounting for 11.8% of the total publications. However, Indonesia's citations are only 173, accounting for 4.3% of the total number of citations, and the citation/publication ratio is 7.2. China owns 14 publications (including 1 MCP article and 13 SCP articles), accounting for 6.9% of the total number of publications, and 152 citations, accounting for 3.7% of the total number of citations, China's citation/publication ratio is 10.9. Israel, Mexico, and Turkey have seven publications (Israel has 7 SCP articles, Mexico has 1 MCP article and 6 SCP articles, and Turkey has 7 SCP articles), accounting for 3.4% of the total publications. However, the number of citations from Israel (45 citations, accounting for 1.1% of the total) and Turkey (46 citations, accounting for 1.1% of the total) is lower than that of Mexico (55 citations, accounting for 1.4% of the total). Brazil has a relatively low number of publications and citations in this data (including 1 MCP article and 5 SCP articles). Germany has the lowest number of publications (5 SCP publications, accounting for 2.5% of the total). Still, it has the highest citation/publication ratio of 17.0, with 85 citations, accounting for 2.1% of the total citations.

Frequently Published Sources

The authors have published 187 sources, including journals, conference proceedings, books, and book series. The most frequently published sources and information on Scopus ranking index, total citation, and local h-index are shown in **Table 3**. The analysis of publication sources reveals the diversity in the selection of journals and proceedings for publishing research on mathematics problem-solving in high school education. The LNCS leads with 12 articles, indicating its popularity among researchers in this field. However, its h-index is only 4, reflecting a relatively moderate impact of the articles published in this source.

Table 3. Top 12 most frequently published sources ranked by number of publications

Rank	Sources (abbreviated)	SQ	CS	TP	h-index
1	Lecture Notes in Computer Science (LNCS)	Q2	2.6	12	4
2	International Journal of Mathematical Education in Science and Technology (IJMEST)	Q2	3.3	11	4
3	Proceedings–Frontiers in education conference, FIE	-	1.1	11	5
4	Journal on Mathematics Education (IndoMS-JME)	Q2	4.2	10	7
5	Journal for Research in Mathematics Education (JRME)	Q1	5.2	8	7
6	Bolema–Mathematics Education Bulletin (BMEB)	Q3	1.0	7	2
7	Educational Studies in Mathematics (ESIM)	Q1	5.6	7	5
8	School Science and Mathematics (SSM)	Q1	2.3	7	4
9	Communications in Computer and Information Science (CCIS)	Q4	1.1	5	1
10	International Journal of Innovation, Creativity and Change (IJICC)	Q3 (2019)	0.5	5	2
11	International Journal of Science and Mathematics Education (IJSME)	Q1	5.1	5	4
12	Journal of Mathematical Behavior (JMB)	Q1	2.7	5	4

Note. SQ: Scopus quartiles; CS: CiteScore 2023; TP: Total publications.

Table 4. Top 11 authors with the most contributions

Rank	Author	Institution/country	TC	TP	TC/TP
1	Santos-Trigo, M.	Centre for Research and Advanced Studies, Mexico	42	6	7.0
2	Rii, P.	Universitas Sriwijaya, Indonesia	74	5	14.8
3	Chiu, M. M.	The Education University of Hong Kong, China	231	4	57.8
4	Zulkardi	Universitas Sriwijaya, Indonesia	53	4	13.3
5	Beal, C. R.	University of Florida, USA	107	3	35.7
6	Camacho-Machín, M.	University of la Laguna, Spain	21	3	7.0
7	Chinnappan, M.	University of South Australia, Australia	80	3	26.7
8	Donnelly, J.	Three Rivers Community College, USA	10	3	3.3
9	Koichu, B.	Weizmann Institute of Science, Israel	27	3	9.0
10	Nathan, M. J.	University of Wisconsin–Madison, USA	121	3	40.3
11	Pontelli, E.	University Las Cruces, USA	31	3	10.3

Note. TC: Total citation, TP: Total publications.

Reputable sources in quartile 1 (Q1) of the Scopus system include the JRME, ESIM, SSM, IJSME, and JMB. Among them, JRME and ESIM stand out with high CiteScores of 5.2 and 5.6, respectively, and impressive h-indices of 7 and 5. These journals demonstrate the significant interest of the academic community in research on mathematics problem-solving. In addition to Q1 journals, the IndoMS-JME (Q2) also makes significant contributions, with 10 articles, a CiteScore of 4.2, and an h-index of 7, highlighting its role in advancing research, particularly in Southeast Asia. Similarly, IJMEST, and FIE, while categorized in Q2 or unranked (FIE), remain popular platforms with many articles (11 each), underscoring their importance in mathematics education. Publication sources in quartile 3 (Q3) and quartile 4 (Q4), such as the BMEB and CCIS, have fewer articles (7 and 5, respectively) and lower h-indices (2 and 1). Nonetheless, they contribute significantly to diversifying the channels for publication. These sources allow researchers to publish studies with narrower scopes or practical applications. In summary, these findings reflect the diverse distribution of research in mathematics problem-solving and highlight the role of reputable publication sources in shaping and advancing research trends. Journals like JRME, ESIM, and IndoMS-JME are key platforms for publishing impactful studies. At the same time, sources such as LNCS and FIE create opportunities for a high volume of publications, thereby contributing to disseminating knowledge in this field.

Contribution by Authors

The total number of authors participating in research and publishing on mathematics problem-solving in high school education is 852. Information about 11 authors with the most publications is shown in **Table 4**. Santos-Trigo, M. is at the top with six publications and 42 citations, reaching a 7.0 publications/citation(s) ratio. Next ranked second is Rii, P. with five publications and 74 citations, a ratio of 14.8 publications/citation(s). Although there are only four publications, Chiu, M. M. achieves the highest number of citations with 231, an impressive ratio of 57.8 publications/citation(s), ranking third. Zulkardi also has four publications but fewer citations, with 53, a 13.3 publications/citation(s) ratio. The remaining authors all have three publications, notably Nathan, M. J. has only three publications but has 121 citations, a high ratio of 40.3 publications/citation(s), Beal, C. R. has 3 publications, 107 citations with a ratio of 35.7 publications/citation(s).

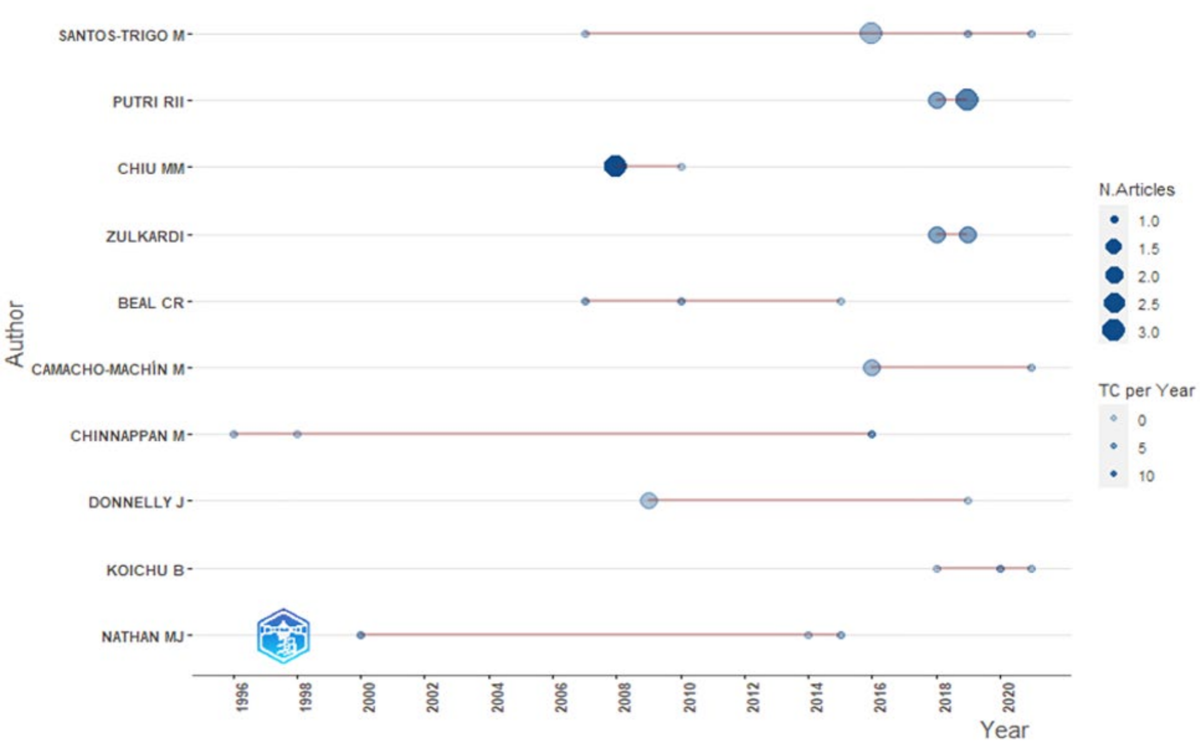

Figure 2. Authors' production over time (Source: Authors' own elaboration)

The publication history of 11 authors over time is shown in **Figure 2**. Santos-Trigo, M. has a reasonably steady publishing frequency between 2007 and 2021, with about 1–3 articles per year. Rii, P. publishes most articles from 2018 to 2019, with about 2 to 3 articles annually. Chiu, M. M. only has about three articles published in 2008. Zulkardi has a reasonably regular publication frequency in 2018–2019, with about two articles per year. Beal, C. R. had a relatively high and frequent publication frequency between 2007 and 2015, with about 1 article per year. Other authors, including Camacho-Machin, M., Chinnappan, M., Donnelly, J., Koichu, B., and Nathan, M. J. have a lower publication frequency, not as frequent as the above authors.

Information About Articles and Citations

Articles in the top 10 are published in various specialized journals such as the Journal of Science Education and Technology, Contemporary Educational Psychology, Journal of Experimental Psychology: Learning, Memory, and Cognition, Journal of the Learning Sciences, etc., showing that these studies have diverse research scopes and subjects (**Table 5**). The total number of citations of these 10 articles is 2240, accounting for about 43.2% of the total collection citations (5,185). This shows that this group of 10 articles is influential and highly cited in the research field. Research topics cover a wide range, from confidence and capacity to solve math problems and knowledge transfer in teaching algebra and physics to the role of software in teaching geometry, etc. This shows that researchers in mathematics education are interested in many different aspects of teaching and learning. Based on the information in **Table 5**, the number of articles with one author is 3 / 10, while the overall ratio is 56 / 334. Thus, the ratio of articles written by one author in this group of 10 articles (3 / 10 = 30%) is significantly higher than the overall ratio (56 / 334 = 16.77%) in the entire data. This shows that these studies have fewer authors than all research projects.

The article "Defining computational thinking for mathematics and science classrooms" by Weintrop et al. (2016) has the highest total number of citations, 767. According to search information on Scopus, it was published in 2015 in Journal of Science Education and Technology, started being cited in 2016, and by 2023, it had 767 citations. The citation process of this article has been outstanding in the past 5 years. It tends to increase gradually each year (2019–2023): in 2019 (91 publications), in 2020 (128 publications), in 2021 (122 publications), and in 2023 (170 publications), showing that research on computational thinking in math and science teaching has received a lot of attention from 2016 to 2021. Regarding content, the article focuses on defining the concept of "computational thinking" and its role in math and science teaching and learning.

Table 5. Top 10 articles with the most citations

CR	Document title	Journal title	TC	TC/year	APA citation
1	Defining computational thinking for mathematics and science classrooms	Journal of Science Education and Technology	767	85.22	Weintrop et al. (2016)
2	Self-efficacy beliefs and general mental ability in mathematical problem-solving	Contemporary Educational Psychology	395	13.17	Pajares and Kranzler (1995)
3	Interdomain transfer between isomorphic topics in algebra and physics	Journal of Experimental Psychology: Learning, Memory, and Cognition	241	6.69	Bassok and Holyoak (1989)
4	Flowing toward correct contributions during group problem solving: A statistical discourse analysis	Journal of the Learning Sciences	148	8.71	Chiu (2008)
5	Gender differences in scholastic aptitude test: Mathematics problem solving among high-ability students	Journal of Educational Psychology	131	4.23	Gallagher and De Lisi (1994)
6	Perceptual learning modules in mathematics: Enhancing students' pattern recognition, structure extraction, and fluency	Topics in Cognitive Science	130	8.67	Kellman et al. (2010)
7	Transfer of domain-specific problem-solving procedures	Journal of Experimental Psychology: Learning, Memory, and Cognition	127	3.63	Bassok (1990)
8	The effects of self-explanation training on students' problem solving in high-school mathematics	Learning and Instruction	109	4.74	Wong et al. (2002)
9	Teachers' and researchers' beliefs about the development of algebraic reasoning	Journal for Research in Mathematics Education	97	3.88	Nathan and Koedinger (2000)
10	The role of a dynamic software program for geometry in the strategies high school mathematics students employ	Journal for Research in Mathematics Education	95	5.28	Hollebrands (2007)

Note. CR: Citation ranking; TC: Total citations.

The authors emphasize the importance of developing computational thinking for real-world problem-solving, data analysis, and modelling. Although the authors do not directly study mathematics problem-solving in the article, they do mention computational thinking as an important skill for understanding and solving math and science problems. Computational thinking includes logical reasoning, data analysis, seeing patterns, breaking problems into small steps, and algorithmizing and automating processes. Therefore, this study contributes to mathematics problem-solving by affirming the importance of computational thinking as an essential skill for complex mathematics problem understanding and solving, which forms a foundation for integrated computational thinking into math teaching to improve students' problem-solving capacities.

Next is the article titled "Self-efficacy beliefs and general mental ability in mathematical problem-solving" by Pajares and Kranzler (1995) with 395 citations, however, the citations are not as focused as the leading publications. head. The lowest is the article "The role of a dynamic software program for geometry in the strategies high school mathematics students employ" by Hollebrands (2007) with 95 citations.

Network Analysis of Keywords Co-Occurrence

The network of keywords is developed from data from articles related to "problem-solving," "mathematics problems," and "high school," including 94 of the most popular keywords, appearing at least 3 times in articles. Synonyms and plurals have been normalized to combine into a single node, the corresponding size being the sum of the component nodes. Each keyword is represented as a node, and the association between keywords is defined by connecting lines with a thickness proportional to the intensity of the relevant association, determined by the number of co-occurrence times. Related keywords are grouped and color-coded. The network is divided into four main clusters represented by four different colors: red, green, blue, and yellow (see **Figure 3**).

The first cluster, in red, includes 34 keywords, some prominent keywords of which are problem-solving, high school, mathematics, mathematics education, algebra, geometry, and metacognition. The research in this group demonstrates the research direction on problem-solving in teaching mathematics in high schools, especially in teaching geometry and algebra. Some typical studies in this group: The role of a dynamic software program for geometry in the strategies high school mathematics students employ (Hollebrands, 2007) mathematical problem posing as a measure of curricular effect on students' learning (Cai et al., 2013);

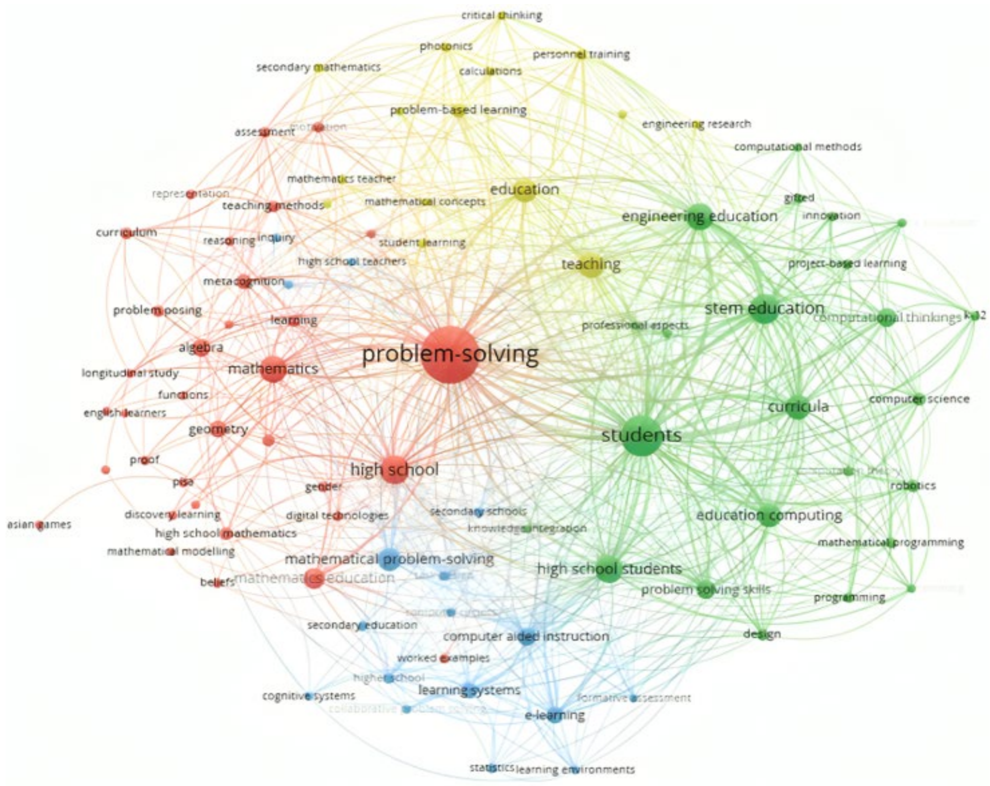

Figure 3. Keyword clustering generated in VOSviewer (Source: Authors' own elaboration)

Teachers' and researchers' beliefs about the development of algebraic reasoning (Nathan & Koedinger, 2000). The green cluster includes 24 keywords related to research on developing problem-solving and computational thinking skills through STEM education, engineering education, and educational computing for students. The main keywords in this group include students, high school students, STEM education, engineering education, curricula, education computing, computational thinking... Some typical studies in this research direction are STEM education design and development (Weintrop et al., 2016); the impact of PBL strategies on STEM knowledge integration and attitudes (Lou et al., 2011). The blue cluster includes 17 keywords, including some main keywords: mathematics problem-solving, learning systems, e-learning, and computer aided instruction ... showing the research trend of using information technology to solve mathematics problems. Some typical studies in this direction are: Schoenfeld's problem-solving theory in a student controlled learning environment (Harskamp & Suhre, 2007); combatting shallow learning in a tutor for geometry problem-solving (Aleven et al., 1998); MATE-BOOSTER: Design of an e-learning course to boost mathematical competence (Barana et al., 2019). The yellow cluster includes 15 keywords; some keywords with the highest frequency are: teaching, education, PBL, and personnel training ... However, these keywords are not distributed centrally in **Figure 3** and do not show any obvious research trends.

Thematic Map

Thematic maps of authors' keywords were generated using the Biblioshiny tool (**Figure 4**). These maps help track the temporal trends of the leading research topics in math problem-solving. Thematic maps allow us to identify the most important research topics and develop and isolate research topics. They also reveal emerging or declining research areas and essential cross-cutting research topics within each interval and across the entire study period. Based on **Figure 4**, the motor themes include the keywords 'computational thinking', 'computer science' and 'K-12', which is an emerging and growing trend. Similarly, the keyword 'K-12' related to basic education also maintained a low occurrence frequency throughout the analysis period, which suggests that trending studies may have focused more on higher education or postgraduate education rather than basic education. However, this trend may change soon, as 'computational thinking' and 'computer science' become increasingly important in modern society, integrating technology into early education can also become a more prominent research topic in the coming years.

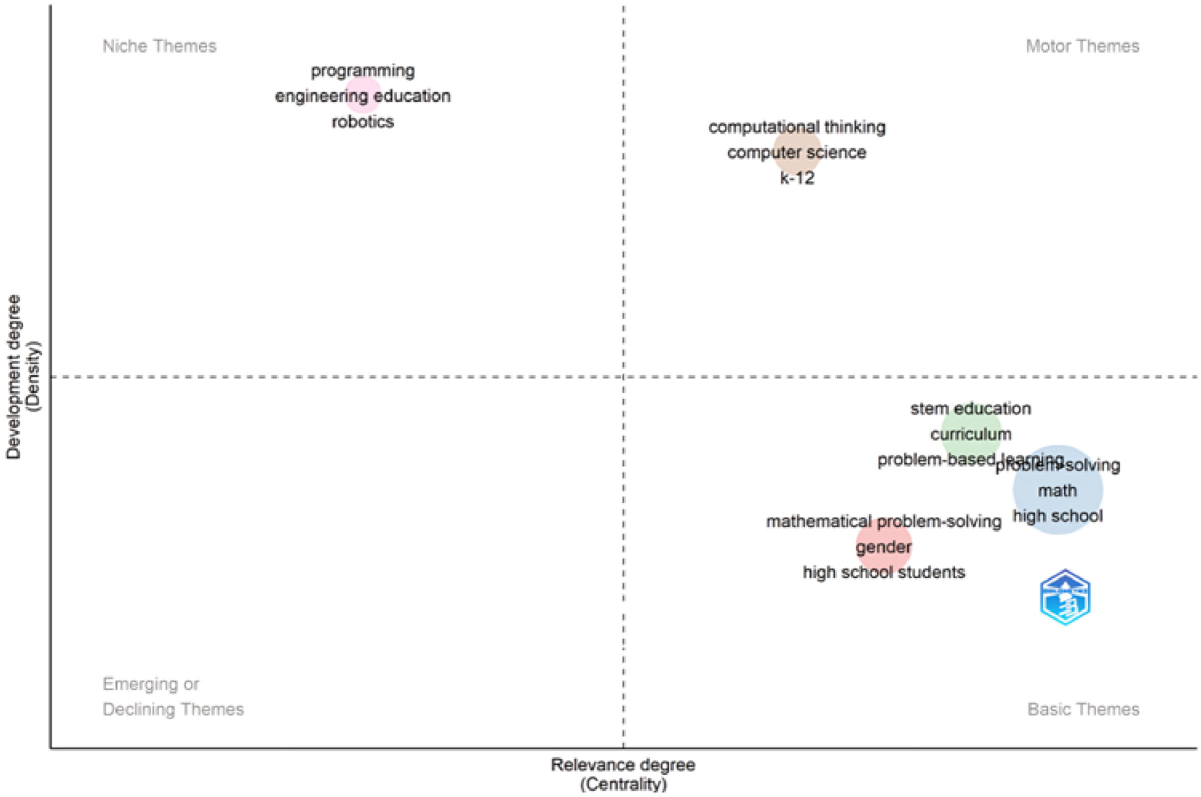

Figure 4. Related thematic maps about math problem-solving (Source: Authors' own elaboration)

The niche themes include keywords such as "programming", "engineering education", "robotics", specific and specialized issues in computer science and engineering, with using robots in teaching programming and engineering that brings many benefits to developing students' problem-solving skills and critical thinking, integrating robotics into engineering and programming education to enhance students' practical skills and learning motivation.

The basic themes include three clusters of keywords, including the first cluster of keywords such as 'curriculum', 'problem-based learning' and 'STEM education', in which the studies focus on developing, evaluating effectiveness, and improving STEM curricula integrated with PBL, as well as surveying the role of teachers and related challenges. The second cluster of keywords including "problem-solving", "math" and "high school", research on methods and strategies for teaching mathematics problem-solving skills to high school students. The third cluster of keywords including "mathematical problem-solving", "gender" and "high school students", the study focuses on surveying and evaluating gender differences in mathematics problem-solving, understanding the affecting factors, developing and evaluating educational interventions to reduce the gender gap, as well as researching effective teaching methods for both male and female students in high school (Gallagher & De Lisi, 1994).

CONCLUSION

This study provides a comprehensive bibliometric analysis of research on mathematics problem-solving in high school education from 1983 to 2023, addressing the key research questions outlined in the introduction.

The analysis reveals a significant increase in publications on mathematics problem-solving in high schools over the past four decades, with an annual growth rate of 7.49%. This growth has been particularly prominent in the last five years, reflecting increasing global interest. However, fluctuations in publication rates suggest evolving research priorities in fields such as artificial intelligence and computational thinking. Among the 36 countries contributing to this field, the USA has the highest number of publications and citations, followed by Indonesia and China. Notable scholars include Santos-Trigo, M., who has the most publications, and Chiu, M.

M., who has achieved the highest citation impact. The study identifies key publication sources, such as Lecture Notes in Computer Science, International Journal of Mathematical Education in Science and Technology, and Journal for Research in Mathematics Education. These journals play a central role in disseminating influential research on the topic. The article "Defining computational thinking for mathematics and science classrooms" by Weintrop et al. (2016) stands out with the highest citation count, underscoring the relevance of computational thinking in mathematics problem-solving. Thematic and network analyses highlight three major research trends: Problem-solving strategies in teaching high school mathematics, especially in geometry and algebra; integrating STEM education and computational thinking to enhance problem-solving skills; and applying information technology in mathematics education to improve teaching and learning outcomes.

In summary, this study highlights the increasing global focus on mathematics problem-solving in high school education, driven by its crucial role in fostering analytical and creative thinking skills. While contributions are unevenly distributed across countries and authors, the findings emphasize international collaboration's importance in advancing this field. Future research should explore effective teaching methodologies, technology's role, and cultural factors that influence addressing the identified gaps and challenges.

This study has several limitations due to the research method characteristics and limitations from input data. We only selected the publications from the Scopus database in English for analysis so that we may have missed some studies in other languages. Although we have tried to remove irrelevant articles, the filtering process may have errors leading to the erroneous removal of publications related to the research topic or irrelevant publications that are still included in the analysis. Some information, such as author names or affiliation names, is not standardized in the Scopus database, which may affect the results we obtained. Finally, information such as authors' research experience and the diversity of their research topics cannot be obtained from Scopus.

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