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Research Article

Argumentation in mathematics and science university textbooks: Similarities and differences in linguistic structures

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ABSTRACT

Argumentation is a key skill in most school subjects and academic disciplines, including mathematics and science. It is possible that similarities and differences between how argumentation is expressed in different subjects can contribute to, or disrupt, students' transferrable argumentation skills. The purpose of this study is therefore to increase the understanding of such similarities and differences concerning the use of argumentation in mathematics and science texts. To reach this goal, the study compares argumentation with a focus on argumentation markers and argumentative structures in first-semester university textbooks in mathematics, chemistry, and biology. Results show that common linguistic argumentation markers in mathematics and science textbooks include for example *because*, *if*, *thus*, *so,* and *therefore* and that there is significantly more argumentation in the mathematics textbook compared to the science textbooks. Further, the results indicate differences in patterns of how argumentation is used, including for example that the mathematics textbook contains more complex argumentation compared with the chemistry textbook. Thereby, the subject-specific languages in the disciplines have the potential to offer students different examples of argumentation.

Keywords: biology, comparative analysis, chemistry, language, linguistic features, reasoning

INTRODUCTION

Argumentation is a key skill in many school subjects and disciplines. In mathematics, reasoning and argumentation are central aspects, both in mathematics as a scientific domain (cf. Aberdein & Dove, 2013) and in school mathematics, according to curriculum documents and frameworks in many countries (e.g., National Council of Teachers of Mathematics [NCTM], 2000; Niss & Højgaard, 2011; Swedish National Agency for Education, 2018). Similarly, argumentation is a central concept within science, both when focusing on scientific reasoning among scientists (cf. Dunbar, 2001; Pera, 1994) and when focusing on science education (e.g., Fischer et al., 2014; Konstantinidou & Macagno, 2013).

Although central to both mathematics and science, argumentation might have different roles within these subjects. Mathematics is often described as being based on deductive reasoning, logic and exact answers, while science is founded on empirical data and experimentally based inductive reasoning (see, e.g., Conner & Kittleson, 2009). Even though these descriptions sometimes are exaggerated and lately more challenged (Develaki, 2020), such differences might contribute to differences between the subjects regarding the type of argumentation that is present and valued, for example, in textbooks. Even though there are many claims in the research literature about differences between mathematics and science, there are very few empirical studies that have examined similarities and differences between texts from these subjects. For example, there are many claims without direct empirical evidence that you need a content-specific type of reading ability for mathematics texts (e.g., Burton & Morgan, 2000; Cowen, 1991; Fuentes, 1998; Konior, 1993; Shanahan & Shanahan, 2008). There are a few studies showing empirical data indicating that there are subject specific reading skills (e.g., regarding mathematical proofs, Inglis & Alcock, 2012), but it is not clear what kind of properties of the texts that could demand the skills. In particular, a literature survey (Österholm & Bergqvist, 2013) shows that there are many claims in research literature about linguistic properties of mathematics texts that distinguish the texts from texts in other disciplines, but that these claims are rarely supported by empirical studies. There are even examples of claims contradicting existing empirical studies (Österholm & Bergqvist, 2013). The need for more empirical data is evident, to test and perhaps challenge the validity of (some of) the many claims about differences between texts from different disciplines.

The purpose of the present study is therefore to increase the understanding of similarities and differences concerning the use of argumentation in science and mathematics texts. We study textbooks in biology, chemistry, and mathematics at university level, since there are generally few studies on argumentation in textbooks, especially at university level but also concerning lower school levels, see Kartika et al. (2021). We delimit our study to the natural language (i.e., written words) used in textbooks, and do not include, for example, symbols and images. The research questions therefore focus on comparing the extent and character of the use of argumentation in natural language in mathematics and science textbooks at university level. The comparisons can show if and how argumentation is used differently and has a different role in mathematics and science education. To identify and characterize argumentation, we use the theoretical concept of argumentative structures (see section theoretical model: argumentative structures).

The results of the present study can be related to both *content literacy* and *disciplinary literacy*. According to Shanahan and Shanahan (2012), content literacy "emphasizes techniques that a novice might use to make sense of a disciplinary text" (p. 8), whereas disciplinary literacy "emphasizes the unique tools that the experts in a discipline use to engage in the work of that discipline" (p. 8). Both concepts relate to students' learning processes, where content literacy focuses on skills needed in the learning process, while disciplinary literacy focuses on goals for the learning process. Thus, the present study will contribute with deeper knowledge of important aspects of the learning processes for mathematics and science, through studying the similarities and differences concerning the use of argumentation, which is an important and fundamental aspect of these content areas.

BACKGROUND

The present study contributes new empirical evidence in relation to two lines of research. One is *comparisons between subjects*, in particular comparisons that can be related to issues of content literacy or disciplinary literacy. The other is *aspects of argumentation* in content areas. For both lines, the present study primarily focuses on the content areas of mathematics and science. Therefore, this background first presents research that compares the language in mathematics and science, and then empirical research that compares aspects of argumentation between the content areas. Finally, the theoretical perspectives adopted in this study are described and the theoretical model used for analysis is defined.

Comparisons Between Language in Mathematics and Science

There are few comparisons of language in mathematics and science. A previous literature review searched for claims in research articles concerning linguistic properties of mathematics texts, and in particular claims comparing mathematics texts with other types of texts (Österholm & Bergqvist, 2013). The survey located 50 references that included claims about linguistic properties of mathematics texts, but only two (Butler et al., 2004; Remmers & Grant, 1928) included empirical data from comparing mathematics texts with texts from another disciplines, and both included science. Further, Ribeck (2015) has contributed to comparisons between mathematics and science texts with an additional empirical study. Another systematic search of literature[1] (in September 2021) revealed two empirical studies in the area, one looking at reading motivation across subjects (Neugebauer & Gilmour, 2020) and one at reading behaviors of experts in different subjects (Shanahan et al., 2011). We conclude that very few empirical studies compare linguistic properties of mathematics and science texts, and none of the studies found includes a direct focus on aspects of argumentation. Therefore, both studies that directly compare linguistic properties of mathematics and science texts, and studies where results can be interpreted indirectly in relation to argumentation, are included in the summary below.

Empirical results show that mathematics texts tend to have a lower level of complexity, when compared to texts from science and other subjects. Remmers and Grant (1928) examined secondary textbooks for many subjects. Mathematics textbooks generally have smaller vocabulary range compared to all other subjects, lower proportion of technical words compared to chemistry textbooks, and of equal magnitude when compared to history textbooks. Butler et al. (2004) examined mathematics, science, and social studies textbooks from grade five. The study showed that mathematics textbooks generally had lower complexity than the other textbooks due to fewer complex sentences and words, lower density, fewer nominalizations, fewer passive constructions, fewer participial modifiers, fewer academic words, and fewer different types of clause connectors. The use of clause connectors could include aspects of argumentation, through the use of connectors such as *because* or, *therefore*. However, since clause connectors include many types of connectors, this result cannot be directly interpreted as concerning argumentation. Ribeck's (2015) analyses included many linguistic properties of secondary textbooks in mathematics and science. One property concerned logical connections, which are more closely related to argumentation than connectors in general. Ribeck (2015) found that mathematics texts contain more logical connections and exclamatory sentences compared to science texts, and that science texts contain more declaration of subject knowledge, and a higher degree of nominal phrases compared to mathematics texts. Taken together, results show linguistic differences between subjects, which can be extrapolated to disciplinary literacy (Shanahan & Shanahan, 2012) and a potential need of content-specific literacy skills (McKenna & Robinson, 1990). In particular, the larger presence of logical connections in mathematics texts indicate that there could be differences in the role or types of argumentation between mathematics and science, where mathematics texts focus more on logical connections.

In contrast to the comparing of texts, Shanahan et al. (2011) compared reading behaviors among experts in mathematics, chemistry, and history. This seems to be the only study that perform a cross-disciplinary analysis of reading behavior. Their results show differences concerning sourcing, contextualization, corroboration, close reading, and rereading, critical response to text, and use of text structure or arrangement and graphics. In line with the differences concerning text properties, the differences in reading behavior indicate relevant differences in disciplinary literacy. However, none of the differences can be directly related to aspects of argumentation.

Argumentation in Mathematics and Science

Argumentation is treated in many different fields of research such as philosophy, linguistics, psychology, and education. In large parts of both science and mathematics education research, the core of argumentation in its simplest form, although formulated in many different ways, is a statement that is backed by a reason (see, e.g., Faize et al., 2017; Kartika et al., 2023). This core is also in line with some everyday meanings of the word, for example, the Britannica Dictionary defines it as "the act or process of giving reasons for or against something" (https://www.britannica.com/dictionary/argumentation). Although most research share this core idea of what argumentation is, the perspectives, the objects of analysis, and the definitions vary both within and between different fields of research (Dove, 2009). For example, variation exists between studies depending on if the analysis focuses on dialogue (Clark et al., 2007), on content and quality of informal arguments (Nussbaum, 2011), or on whole-class and small-group student discussions (Erduran et al., 2004).

[1] In ERIC, only academic journals in English. Several different searches: In Title all combinations of: ling*/lang*/text* AND math* AND sci*/biolog*/chem* (no relevant results) In Abstract, ling* AND math* AND sci*

In addition, other similar or overlapping notions are sometimes used, such as reasoning and justification. In the present study, we use a theoretical model called *argumentative structures* (presented more closely in the section theoretical model: argumentative structures) that aims to capture this core of argumentation, of giving reasons for or against something. The present study concerns argumentation in mathematics and science textbooks, and in line with this we delimit this section to studies focusing on *presentations* of argumentation, and explicit properties of argumentation.

Quantitative characterizations of argumentation in school presentations, primarily in textbooks, show that the use of argumentation is not very common. This general conclusion exists in studies that analyze mathematics textbooks from primary to upper secondary school (Davis, 2012; Newton & Newton, 2006a; Stacey & Vincent, 2009; Thompson et al., 2012; Österholm & Bergqvist, 2013), in studies focusing on science textbooks in primary school (Newton et al., 2002), and in studies on teacher discourse in primary science classrooms (Driver et al., 2000). The conclusion also seems to concern other subjects than mathematics and science. Newton et al. (2002) and Newton and Newton (2006c) do not make a direct comparison between subjects, but by using the same methods, they draw conclusions about the lack of argumentation in textbooks in mathematics (Newton & Newton, 2006a) and science (Newton et al., 2002), as well as in music (Newton & Newton, 2006b) and religion (Newton & Newton, 2006c).

Only three studies that perform a direct comparison of argumentation between subjects where either mathematics or science is included have been found. Firstly, Triantafillou et al. (2016) compared school textbooks for mathematics and science, concerning the topic of periodicity in both subjects. They analyzed how different modes of reasoning are used, which distinguishes what reasoning is based on, in particular separating between empirical, logical, and mathematical. The differences in the textbooks can be said to be in line with more general properties of the subjects, since the physics textbook mainly rely on experimental evidence while the mathematics textbook rely on mathematical evidence. Secondly, Österholm and Bergqvist (2013) compared mathematics and history textbooks concerning the number of logical relations (e.g., because and therefore) and temporal relations (e.g., thereafter or before). The mathematics texts included fewer logical and fewer temporal relations compared to the history texts, but there were not enough relations to perform statistical analysis. If mathematics is seen as focusing more on logic and deduction, this result seems somewhat counter-intuitive. Thirdly, Herrenkohl and Cornelius (2013) analyzed argumentative practices in science and history classrooms where the same teacher taught both subjects. They identified different practices, such as *formulating arguments* and *constructing alternative arguments* and found that changing and revising arguments was more common in science, and imaginative and analogous thinking was more common in history. The difference in imaginative and analogous thinking could be related to basic characteristics of the subjects, but the difference in changing and revising arguments does not clearly relate to properties of the subjects.

In both cases—when it is easy, or not so easy, to give logical reasons for differences between subjects based on theoretical properties—the differences may concern disciplinary literacy, at least in some context or for some educational level. We argue for a need to combine empirical comparisons between subjects with theoretical considerations concerning differences between disciplines. Based on theoretical considerations, certain hypotheses of possible differences between texts in mathematics, chemistry, and biology can be formulated. For example, based on the characterization of mathematics as more deductive and logical, and science as more inductive and empirical, it can be seen as reasonable that mathematics texts contain more instances of argumentation and also perhaps more complex instances of argumentation. In general, any types of differences between texts from different subjects found in empirical data have the potential to be interpreted in relation to properties of the subjects. Thereby, besides contributing with empirical knowledge of differences between texts from different subjects, this study can also contribute with more in-depth understanding of properties of different subjects.

Theoretical Model: Argumentative Structures

A limitation in much research on argumentation is that there is no explicit method for locating instances of argumentation, that is, for identifying the concrete parts of communication that determines whether it can be classified as argumentation or not. Some studies have used an explicit method for locating argumentation, but then only focused on specific types of markers, such as particular types of words. For example, Newton p → **c**

Figure 1. The model for argumentative structures consists of three parts: **conclusion** (**c**), premise (p), and argumentation marker (→) (in the present paper, the **conclusion** is always marked with bold and the *argumentation marker* with italics in all examples) (Source: Authors)

and Newton (2006c) focus on “clauses of cause (typically signaled by words like as, because, and since) and purpose (typically signaled by in order to, to, and so that)” (p. 73).

According to a content analysis of articles on argumentation in science and mathematics education, Toulmin’s (1958) model is the most commonly used (Kartika et al., 2021). Toulmin (1958) model includes the concepts of claim, data, and warrant, where a claim is a statement de-rived from the data in accordance with a principle or a warrant. There are many other models, but in the core of all of them are statements that are supported or argued for, that is, the claim or conclusion, and other statements that support these conclusions, that is, the premise(s). A problematic issue with the Toulmin (1958) model, as well as with many of the more complex models, is that there is a built-in ambiguity. For example, Toulmin (1958) acknowledges (in the original publication) that although it is sometimes possible to clearly separate between data and warrant, it is not always possible (p. 92). This ambiguity is also visible in empirical studies (Erduran et al., 2004; Osborne et al., 2004; Rasmussen et al., 2015).

In the present study, we use a theoretical model that captures *argumentative structures*. The model describes situations when *something is brought forward as support or reason for a statement* and consists of three parts: conclusion, premise, and argumentation marker, see **Figure 1**. Different versions of this model are established within previous research (Freeman, 1991), but we put more weight on the argumentation marker than most other models do, since the marker is often the key component to identify the argumentative structure.

Looking at the parts of the model, the conclusion (**c**) is a statement or a proposition, that is, “an expression in language or signs of something that can be believed, doubted, or denied or is either true or false” (Merriam-Webster/proposition, 2a). Such a statement is given in the form of a declarative sentence, which means that it has to include a subject and a verb (e.g., https://www.grammarbook.com/blog/clauses-sentences/declarative-sentences/). This excludes questions (interrogative sentences), exclamations (exclamatory sentences), and commands (imperative sentences). Simply put, the conclusion is a declarative sentence that “declares” something. The verb can be in the form of a word, as in the example “the flower is red”, or in the form of a symbol or gesture or similar, as in the example “$f(x) = x + 3$” where the equal sign can be read “equals”. The premise (p) is that which is brought forward as support or reason for the conclusion. It is often also in the form of one or several declarative sentences.

The argumentation marker (→) is something that signals that the premise is given as support or reason for the conclusion. From pilot studies, we have found that these markers can (in written text) exist in the form of specific words or expressions (sometimes combined with layout), grammatical forms, or symbols. For many of the categories, options vary with language (e.g., English or Swedish). In the present study, we focus only on the natural language (i.e., written words) in English textbooks, so the argumentation markers can be words, phrases, or grammatical structures. We denote these *linguistic argumentation markers*.

Note that the order of the premise, the argumentation marker, and the conclusion can vary. Depending on the grammatical structure and the argumentation marker, the conclusion can be placed before or after a premise and sometimes both options are equally correct. Similarly, sentences can be constructed with argumentation markers in different places in relation to the premise and/or the conclusion.

Returning to the argumentative structure, there is a direction from the premise towards the conclusion, and a relation between them, but not necessarily a strict logical or cause/effect relation. Some argumentation markers clearly signal logical connections or logical argumentation (e.g., ‘therefore’, ‘⇒’), while others can be used to signal other connections (e.g., ‘so’, ‘when’). An example of a sentence including an argumentative structure in a chemistry textbook is “There are three pairs of electrons around the C atom; *therefore*, **the electron pair arrangement is trigonal planar**”. The argumentative structure can be visualized, as in **Figure 2**.

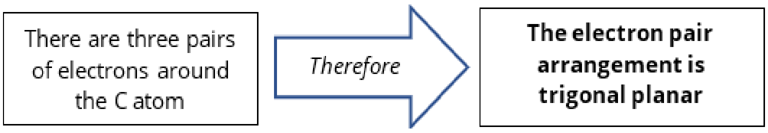

Figure 2. Example of an argumentative structure from chemistry with premise, argumentation marker and conclusion (Source: Authors, text example from Chang, 2010)

Argumentative structures can be much more complex than premise and conclusion connected by an argumentation marker. For example, a conclusion can be the premise for another conclusion in a chain. Still, the theoretical model presented here makes it possible to identify, quantify and compare the argumentative structures in texts from different disciplines.

The operationalized method enables us to learn more about the qualitative features of argumentation in natural language by studying a broad range of connections between statements.

PURPOSE AND RESEARCH QUESTIONS

The purpose of this study is to increase the understanding of similarities and differences concerning the use of argumentation in science and mathematics texts. This is approached by comparing linguistic argumentative structures in first-semester university textbooks in mathematics, chemistry, and biology. The research questions are, as follows:

1. What are the most common linguistic argumentation markers in mathematics, chemistry, and biology?
2. How common are linguistic argumentative structures in mathematics, chemistry, and biology, and are there any differences between the disciplines?
3. What are the similarities and differences between mathematics, chemistry, and biology regarding the main characteristics of argumentative structures?

While the first research question explores and compares the use of argumentative *markers*, the second question compares the general commonness of argumentative *structures* in the different disciplines. The third research question compares the main characteristics of the argumentative structures between the different disciplines, based on the theoretical model and how its manifestations can vary. This means that we examine whether the markers mostly mark the premise or the conclusion, the order of the premise and conclusion, and the complexity of the argumentative structures.

METHOD

The research questions concern similarities and differences between mathematics, chemistry, and biology textbooks at university level, regarding commonness and characteristics of different aspects of linguistic argumentative structures. The analysis was carried out in four steps. First, argumentative structures in three textbooks (one for each subject) were identified and recorded. Second, the most common argumentation markers were identified, both in total and in mathematics, chemistry, and biology separately. Third, the argumentative structures were counted for each textbook and statistical methods were used to compare the disciplines. Fourth, characteristics of the argumentative structures were identified and their use in the different disciplines compared. In the following subsections, we present the selection of data and the four steps of the analysis in more detail.

Selection of Data

In Swedish university courses in mathematics and science, it is common with international literature written in English. The reasons are mainly that the target group is too narrow for producing course books in Swedish, and that most Swedish students are good at reading English. In our study, we therefore select textbooks in English that are also used in other countries, which also makes the results more relevant internationally.

One textbook each in mathematics, chemistry, and biology was selected to represent what a student can meet at a Swedish first semester university course. The textbooks were chosen according to the following criteria: the book is

(a) written in English,

(b) used in Swedish basic level university courses, and

(c) also used internationally.

The selected books were Campbell biology (Reece et al., 2013), Chemistry (Chang, 2010), and Calculus: A complete course (Adams & Essex, 2013). From each book, chapters representing five weeks full time studies (7.5 ECTS) were selected based on authentic schedules and reading instructions from Swedish universities. For each book, 20 pages from the included chapters were randomly selected for analysis, thereby representing the first-semester five-week course content. Pages where 75% or more consisted of figures, exercises, or similar (other than text) were excluded from the analysis. If one such page was randomly selected, it was ignored, and a new page was randomly selected. Thereby, all selected pages contained at least 25% text and/or examples.

Step 1: Identifying Argumentative Structures in the Data

As described in the Theoretical model, the argumentative structures consist of conclusions, premises, and argumentation markers. Since we delimit to linguistic argumentative structures, we only include written words in our analyses, and not symbols and images. To identify all linguistic argumentative structures in the text, firstly, each statement in the form of a declarative sentence was identified. Declarative sentences are full sentences that are not questions, exclamations, or commands. Secondly, the statements and the text surrounding them were scanned for argumentation markers linking other part of the text to the sentence, in such a way that *something is brought forward as support or reason for the statement.* If such links can be found, the statement is a conclusion and the parts of the text that supports it is the premise(s).

The key component in identifying argumentative structures is the argumentation markers and in the present study we focus only on linguistic markers. According to the definition of argumentative structures, the argumentation marker is something that signals that the premise is given as *support or reason* for the conclusion. Consequently, we include both words, phrases, and grammatical structures, that clearly signal a logical connection between statements, but also those that more vaguely connects statements in this way. Linguistic markers that more clearly signal a logical relation are usually words that in a dictionary are defined as referring to cause or reason, as their main meaning. These include, for example, *since*, *because*, *thus*, *therefore*, *if … then*, and *hence*. Others are words and phrases that *can* be used to signal a logical property between statements, but that also are used in many other ways. These include for example: *that is*, *according to*, *consistent with*, *as*, *when*, and *which is why*. Each time an argumentative structure was identified, it was recorded for further analysis guided by the research questions. When a previously unknown argumentation marker was identified, the dataset was scanned again for similar cases.

Step 2: Identifying the Most Common Argumentation Markers

The first research question concerns identifying and characterizing the most common linguistic argumentation markers in mathematics, chemistry, and biology. The occurrence of different argumentation markers was summarized for the full dataset as well as for mathematics, chemistry, and biology individually. To give a clearer understanding of how argumentative structures are built with the markers, we highlight some examples from the most common as well as from uncommon argumentation markers.

Step 3: Counting and Comparing Occurrences of Argumentative Structures

The second research question concerns how common argumentative structures are in mathematics, chemistry, and biology. This question is answered by using a quantitative approach. To get a comparable measure of argumentative structures between the books, through the randomly selected 20 pages that contained various amount of text per page, we calculated *the number of argumentative structures per declarative sentence*. The calculations demanded two measurements: *the number of declarative sentences* on each page and *number of argumentative structures* on each page. First, we tallied the *number of declarative* *sentences* on each page. Initially, all complete sentences on each page were identified. Sentences that started or ended on another page were excluded. In addition, since a declarative sentence is a sentence that makes a statement, all sentences that were commands, questions, or exclamations were excluded. Second, we tallied the *number of argumentative structures* for each page as described in step 1. After this, the *number of argumentative structures* was divided by the *number of declarative sentences* for each of the 20 pages from each book.

To compare how common argumentative structures are in mathematics, chemistry, and biology, a one-way between-groups analysis of variance (ANOVA) was conducted using SPSS 28. The analysis treated the 20 results from the individual pages of each book as repeated measurements capturing variations in layout and amount of text that is characteristic for each discipline. In the ANOVA, the homogeneity assumption was violated, since the mean varied significantly across the groups, so the Welch-statistic is reported and the Games-Howell post-hoc test was used to find where there is a difference between groups.

Step 4: Comparing Characteristics of Argumentative Structures

The first two research questions concern the choice of argumentation marker and how often such markers are used within the disciplines. As mentioned when the theoretical model was presented, there are some other central aspects that can vary when an argumentative structure is constructed. The third research question therefore concerns similarities and differences between the disciplines regarding the following three aspects.

The first aspect is if the *argumentation markers mark the premise or the conclusion*. Some argumentation markers, like *because*, focus on the premise, in sentences like "**C** *because* P", or even "*Because* of P, …". Other argumentation markers, like *therefore,* focus on the conclusion, in sentences like "P *therefore* **C**" or simply "*Therefore*, **C**", where it can be unclear what is seen as the premise. The second aspect is *whether the premise or the conclusion is presented first*. For example, "She ran faster, *therefore* **she won**." (p – **c**) or "**She won** *because* she ran faster" (**c** – p). The third aspect concerns *if the argumentative structure is combined with other argumentative structures in more or less complex compositions*. For example, the simplest form consists only of a premise, a conclusion, and a marker, not connected to any other structure, just as many of the above examples show. A more complex situation is when two simple structures are combined, for example, (p → **c**) → **c**. In words, this could be "When he runs, he is more likely to fall. Therefore, he mostly walks." An argumentative structure is considered to be involved in a complex composition when two or more markers are used to connect two or more argumentative structures. The differences between the mathematics, chemistry, and biology textbooks regarding these three characteristics are described and exemplified, and the quantitative differences are statistically analyzed using Kruskal Wallis Tests in SPSS 28.

RESULTS

This section is arranged in accordance with the research questions. With a focus on comparing mathematics, chemistry, and biology, we start with argumentation *markers*, move to argumentative *structures*, and finish with characteristics when constructing argumentation. Finally, research questions are answered.

Common and Uncommon Argumentation Markers

The most common linguistic argumentation markers, defined as those that occur four times or more in the sample from each book, make up 70-82% of all markers found. Overall, the most common argumentation marker is *because,* which occurs 62 times in the dataset. The second most common marker is *if*, which occurs 51 times, of which 30 times alone, and the third most common is *thus* (25 times). **Table 1** shows the most common markers for the mathematics, chemistry, and biology textbooks as well as the overall most common markers.

Because is the most common argumentation marker, and one example[2] of the use in chemistry is "*Because* of their great reactivity, **the halogens are never found in the elemental form in nature"**. *If* is the second most common marker, sometimes used alone and sometimes in combination with *then*. An example from

[2] The **conclusion** is marked with bold and the *argumentation marker* with italics in all examples.

Table 1. Common (i.e., occurring at least 4 times) argumentation markers in mathematics, chemistry, biology, and in the subjects combined

	Markers in mathematics	Markers in chemistry	Markers in biology	Markers in all subjects combined
	if – then (16) so (13) thus (13) since (12) if (10) therefore (9) hence (7) as (5) then (5)	because (38) thus (6) therefore (6) if (5) so (5) according to (5) if – then (4)	because (21) if (15) when (12) [ing] (9) thus (6) causes (5) by [ing] (5) as (4)	because (62) if (30) thus (25) if – then (21) so (20) therefore (17) when (15) since (14) as (12) hence (7)
Total number of common markers	90	69	77	-
% of markers that are common	82	70	73	-

Note: The number of times the marker occurs is shown in parenthesis, for example, the marker *because* occurs 38 times in the chemistry data. Each column is ranked in order of occurrence and summarizes the number of common markers and calculates how many percent of the total number of markers they make up. For example, in mathematics 82% of the argumentation markers are common.

Table 2. Number of declarative sentences, number of argumentative structures, and number of argumentative structures per sentence in mathematics, chemistry, and biology, respectively

	Number of declarative sentences (total)	Number of argumentative structures (total)	Argumentative structures per sentence (mean from independent pages)
Mathematics	259	110	0.46
Chemistry	363	102	0.27
Biology	590	108	0.18

Note: The mean number of argumentative structures per declarative sentence is calculated from the sample of 20 independent pages from each book.

mathematics is "*If* Q is a point on C different from P, *then* **the line through P and Q is called a secant line to the curve**". *Thus* is the third most common argumentation marker. An example from the biology textbook is "The resulting H^+ gradient represents potential energy that can be used for active transport–of sucrose, in this case. *Thus,* **ATP indirectly provides the energy necessary for cotransport**". In summary, most of the most common argumentation markers are linguistically conjunctions (e.g., *because*, *if*, *since*, and *as*) and adverbs (e.g., *thus*, *hence*, *therefore*, *then*, and *so*).

The remaining 20-30% of the argumentation markers are considered less common (i.e., occurring fewer than four times each). They include words or expressions as well as a grammatical structure. Some of the words are prepositions, for example, *according to, by, due to,* and *via*. Some are verbs that by their meaning point out a conclusion, for example, *causes, concludes, shows, results, means, reveals,* and *leads (to)*. The expressions have either a clear and explicit meaning, for example, *with the result that, which is why* and *is attributable to*, or are less certain, such as *these … underscore* and *suggested that*. Finally, one grammatical structure was classified as an argumentation marker. This was verbs in -ing-forms such as *causing,* relatively commonly found in the biology textbook. An example of this from the biology textbook is: "Auxin also rapidly alters gene expression, *causing* **cells in the region of elongation to produce new proteins within minutes**". No other type of grammatical structures was identified as argumentation markers in the data. Taken together, the less common argumentation markers can be words, expressions, or grammatical structures of various types.

Commonness of Argumentative Structures in Different Disciplines

The second research question concerns how common argumentative structures are in mathematics, chemistry, and biology textbooks. This quantitative comparison aims to show how much argumentation is used in the each of the different textbooks. The final dataset included 320 argumentative structures. The textbooks contain 0.18 (biology), 0.27 (chemistry), and 0.46 (mathematics) argumentative structures per declarative sentence, see **Table 2**.

Table 3. Number of times premise and conclusion is marked in the data from each book

	Marking premise	Not marking premise	Marking conclusion	Not marking conclusion
Mathematics	57	53	66	44
Chemistry	74	28	34	68
Biology	69	39	37	71

Note: Some markers mark both premise and conclusion.

Table 4. The five most common markers in mathematics, chemistry, and biology, sorted based on whether they mark premise, conclusion, or both (*)

	Markers of premise	Markers of conclusion
Mathematics	if – then* (15%) since (11%) if (9%)	if – then* (15%) so (12%) thus (12%)
Chemistry	because (37%) if (5%)	thus (6%) therefore (6%) so (5%)
Biology	because (19%) if (14%) when (11%) [ing] (8%)	thus (5%)

Note: For each marker, the percentage of markers from a textbook that consist of this particular marker is noted in parenthesis.

The one-way between-groups ANOVA showed that there was a statistically significant difference at the $p < 0.05$ level in the use of explicit argumentative structures between the books. The mean differs significantly across the groups [F_{Welch} (2, 36) = 9.6, p = 0.001]. Post-hoc comparisons using the Games-Howell test indicated that the mean value for biology (mean [M] = 0.18, standard deviation [SD] = 0.12) was significantly different from mathematics (M = 0.46, SD = 0.27), that the mean value for chemistry (M = 0.27, SD = 0.14) was significantly different from mathematics, and that the mean values for biology and chemistry did not differ significantly.

Characteristics of Argumentative Structures in Different Disciplines

The third research question concerns the characteristics of the argumentative structures and potential differences between the disciplines. Three central aspects are compared: if the argumentation marker marks the premise or the conclusion, the order of premise and conclusion, and the complexity of the argumentative structures.

Markers of premise or conclusion

There is a statistical difference between the mathematics and science books concerning whether it is more common that premise or conclusion is marked. The conclusion is more often marked in mathematics textbooks and the premise is more often marked in the science textbooks. Two Kruskal-Wallis tests, one for marking of premise and one for marking of conclusion, were performed and both show that there are significant differences between the textbooks. For marking of premise, $X^2(2, N = 320) = 9.81$, p = .007 and for marking of conclusion, $X^2(2, N = 320) = 20.18$, p = .000. Post-hoc analysis with Bonferroni adjustment correcting for multiple tests shows that there are significant differences between mathematics and chemistry in terms of marking premise and significant differences between mathematics and chemistry as well as biology in terms of marking conclusion (**Table 3**).

Looking at the five most common markers from each textbook (cf. **Table 1**), **Table 4** reveals more details about the variations in the natural language within and between the textbooks. For example, in mathematics, the three most frequently used markers all mark conclusions, although *if – then* (the most common), also marks the premise. This is in line with the quantitative result above. Similarly, in biology, the four most common markers all mark premises. In chemistry, the most common marker, *because*, marks premise, while the following two mark conclusions. However, since *because* constitutes 37% of all markers found in the chemistry textbook, marker of premise is clearly dominant.

Table 5. Number of argumentative structures with the order of premise-conclusion (P-C) and conclusion-premise (C-P) in the data from mathematics, chemistry, and biology textbooks

	P-C	C-P
Mathematics	94 (85%)	16 (15%)
Chemistry	81 (79%)	21 (21%)
Biology	82 (76%)	26 (24%)

Table 6. Number of complex and non-complex argumentative structures in the data from mathematics, chemistry, and biology textbooks

	Complex	Non-complex
Mathematics	36 (33%)	74 (67%)
Chemistry	18 (18%)	84 (82%)
Biology	21 (19%)	87 (81%)

Order of premise and conclusion

The order premise-conclusion was more common than the order conclusion-premise for all textbooks, see **Table 5**. Also, a Kruskal-Wallis Test showed the three books were not significantly different from each other in terms of order of premise and conclusion, $X^2(2, N = 320) = 3.20$, $p = .20$. Instead, all show a similar pattern.

All examples given before have the more common order premise-conclusion. One example of the opposite, the conclusion preceding the premise, is **"Hydrolysis of glycogen in these cells releases glucose** *when* the demand for sugar increases".

Complexity of argumentative structures

An argumentative structure is considered complex when two or more markers were used to connect two or more argumentative structures. Complex argumentative structures were most common in the mathematics textbook (see **Table 6**). A Kruskal-Wallis test revealed a statistical difference between the textbooks in terms of complex argumentative structures, $X^2(2, N = 320) = 8.13$, $p = .017$. Post-hoc analysis with Bonferroni adjustment correcting for multiple tests shows that the significant difference is between mathematics and chemistry.

One example of a complex structure from the chemistry textbook is: *When* the cell is stimulated, **gated channels open** *that* **facilitate Na^+ diffusion**. In this sentence, the argumentation markers are *when* and *that*. The conclusions are "**gated channels open**" and "**facilitate Na^+ diffusion**". The premises are "the cell is stimulated" and, for the second argumentative structure, "gated channels open". Thus, "gated channels open" serves as conclusion for the first argumentative structure and as premise for the second, at the same time. In the data, this is counted as two argumentative structures and defined as complex.

Summary of the Results

Based on the presented results, we can provide answers to our three research questions:

1. We can conclude that common linguistic argumentation markers in biology, chemistry, and mathematics textbooks include, for example, *because*, *if*, *thus*, *so,* and *therefore,* and that the pattern of occurrence differs between the disciplines.
2. The three university textbooks contain 0.2 (biology) to 0.5 (mathematics) argumentative structures per declarative sentence. With much simplification, this can be understood as if one paragraph in the biology textbook consists of ten sentences, it includes in average two argumentative structures, while in the mathematics textbook it includes in average five argumentative structures. The difference between mathematics textbooks and the science textbooks is statistically significant.
3. Comparing characteristics of argumentative structures showed two differences. Firstly, the argumentation marker is more often marking the conclusion in mathematics and more often marking the premise in science. Secondly, the argumentative structures are more complex in the mathematics textbook compared to the chemistry textbook.

DISCUSSION

Through this study, we have learned more about similarities and differences concerning the use of argumentation in science and mathematics textbooks. Thereby, we have added valuable empirical evidence, since comparative analyses of texts in science and mathematics are missing in previous research. Thus, our empirical analyses contribute to a more fact-based starting point in discussions about differences between the disciplines, which can relate to both content literacy and disciplinary literacy. First, discussions around differences between the domains can focus on the potential need for content-specific literacy skills, that is, to highlight if and how students might need different skills and strategies when encountering texts from different disciplines. Second, our results can also enable in-depth discussion of more general differences between disciplines, including aspects of disciplinary literacy. These issues are discussed more below.

Some of our results can be connected to more general properties of the disciplines. In particular, mathematics in relation to science can be seen as more deductive in nature and focusing more on logical relations. Based on this, it is reasonable that mathematics texts contain more argumentative structures and also more complex argumentative structures as our results show. Concerning the amount of argumentation, the results are in line with the study by Ribeck (2015), analyzing textbooks at secondary level. The result that argumentation is more frequent in mathematics than in science can therefore be a more general pattern. Further, if mathematics is seen as more deductive in nature, it is reasonable that premises more often precede the conclusions. The data shows a tendency that the order premise-conclusion indeed is more common in mathematics, but the difference is not statistically significant. In addition, the results show that it is a common feature for both mathematics and science that the order premise-conclusion is more common than the order conclusion-premise. That is, there are more similarities than differences between the subjects concerning the order between premise and conclusion. The result that the argumentation marker signals the conclusion more often in mathematics, and more often signals the premise in science, can perhaps also be related to the more deductive nature of mathematics. When comparing mathematics and science, as done above, we can draw the conclusion that argumentation seems to play somewhat different roles in these subjects, and that this can, at least partly, be related to more general properties of the disciplines.

Above, we have explored potential connections between properties of texts from different subjects and more general properties of the disciplines. However, the differences between the texts could also depend on other things, and a critical question is if the texts are representative. For example, the different textbooks have different authors, who might have different styles of writing, including how to present argumentations. Unfortunately, the only similar previous research we have is the one study from Ribeck (2015). Even if her result is similar to ours, argumentation is more frequent in mathematics textbooks compared to science textbooks, this is not sufficient to draw general conclusions. Neither is our results, or our results together with previous research, enough to make clear generalizations concerning properties of textbooks from the different subjects or language used in different disciplines. Thus, we see the need for more empirical studies that compare texts to be able to draw more reliable conclusions concerning differences between subjects. Such knowledge can be helpful in understanding potential differences in the learning processes for students, concerning issues in relation to content literacy and disciplinary literacy (cf. Shanahan & Shanahan, 2012).

Although we cannot easily generalize our results, they address important aspects of students' learning processes, in relation to scientific and mathematical argumentation, which are important parts of content literacy and disciplinary literacy. The empirical data was selected to allow for ecological validity, that is, our textbooks were chosen based on what a student at a Swedish university could encounter during their first-semester studies. Therefore, the differences we have seen between the subjects can be experienced by students, which then could highlight patterns and possible issues of content and disciplinary literacy they meet. In addition to teachers and disciplinary experts, textbooks can function as role models, where texts reflect different structures of argumentation in the different subjects through the written language. Based on these differences, the meaning of argumentation in science and mathematics might differ, at least as experienced by the first-semester students who use these textbooks. As a consequence, students might need to adapt their study behavior and learning process for successful outcomes in different courses, that is, they need to develop aspects of content literacy and disciplinary literacy in parallel.

Besides the empirical results our study has produced, we have argued for the need, and demonstrated the usefulness of, more operationalized constructs in empirical analyses of argumentation. In particular, we have used argumentative structures, where focus is on the explicit use of argumentation markers. This is a delimitation to the more explicit types of argumentation but allows for more in-depth comparative analyses between texts from different disciplines. Through our analyses of the amount and properties of argumentative structures, we have shown some clear differences between university textbooks from mathematics and science. We have used these results as a basis for discussing the potential for more general differences between the disciplines. The results can also be seen as important pre-conditions for, as well as integral parts of, students' learning processes in these subjects. Therefore, the results can be used as a basis for empirical studies that focus on comparative analyses of students' learning processes in different subjects.

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