Analysis of analogies in Turkish elementary mathematics textbooks

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Abstract
The aim of this study is to determine analogies that are used in Turkish secondary school mathematics textbooks. According to this aim, 7 textbooks are analysed: 3 of them are published by the commission of Ministry of Education and 4 of them are published by private publishing companies. The document analysis method is used in the study. Analogies are classified with respect to their analogical relationship, presentational format, abstraction, position of analogue with respect to target and level of wealth. Classified analogies were examined separately within the relevant textbooks and added to the findings by specifying the total number of analogies. Then the data were analyzed and reported. As a result of the research, it was found that analogies are dominantly structural, verbal-pictoral, preliminary and simple.

Keywords: Analogies, mathematics education, elementary mathematics textbooks.

Introduction
Permanent changes in behavior as a result of individuals’ interaction with the environment are defined as learning. There are many theories about how learning has come about. The most popular of these are theories developed by Piaget, Bruner, Gagné and Ausubel (Özmen, 2004). In Ausubel’s pioneering “Meaningful Learning” approach, it is necessary for learners to attach meaning to newly learned knowledge, to associate and integrate new knowledge with prior knowledge (Kara and Özgün-Koca, 2004). Due to the accumulation structure of mathematics, the present concepts and knowledge of the learner are steps for newcomers ones (Küçük and Demir, 2009). Analogy is a method that can be used to create relationships between these concepts.

Analogy is a comparison between the similarities of two concepts (Glynn, 2008). Analogies aim to help learners better understand the new concept by associating attributes between known (old) and unknown (new) concepts (Spezzini, 2010). Analogies are important in understanding the concepts that are abstract in nature due to the nature of mathematics (Guo and Pang, 2011; Rittle-Johnson, Star and Durkin, 2009; Richland, Zur and Holyoak, 2007; Mazur and Pesci, 2005; Richland, Holypak and Stigler, 2004). The use of analogies in mathematics education has been shown to be effective in terms of students’ concepts.

Textbooks are basic educational elements that explain the information belonging to the subjects in the relevant curriculum in a certain order and plan, and direct the students and teachers towards their achievements (Ünsal ve Gunes, 2004). At the same time, textbooks are the most used tools in educational activities (Demirel, 2016). For this reason, it is expected that analogies will be included in textbooks. At the same time, in order to determine how analogies support learning in textbooks, the analogies in the books need to be uncovered and classified (Çalık and Kaya, 2012).
As expressed previously, mathematics is based on the relationship between previous knowledge and new knowledge. As an aim of mathematics education, it is not only that learners develop mathematical skills or increase their mathematical knowledge. In addition to that, they are also good problem solvers and also to see connections between different events.

In order to understand the abstract concepts that nature of mathematics requires, they need to be concretized. The abstract mathematical concepts are made use of modeling, stories and analogies. Through analogies, one of the teaching techniques, it is aimed to explain a concept that the students cannot understand by starting from a concept that they have learned beforehand. It is stated that the analogies are persistent in the minds of the students and that new concepts which are desired to be taught in this context can be remembered for a long time (Thiele and Tregast, 1994).

It is aimed to determine what types of analogies are included in the elementary school mathematics textbooks in the direction of the aim of the study and to determine the points of view about the analogy of the books by investigating whether these kinds of analogies differ according to the printing houses. It is considered that the information about the analogies in the textbooks examined and the results to be obtained are references for new researches to be made in the future, and that there is a lack of knowledge in the field. At the same time, it is expected that the mathematics textbooks prepared in Turkey will be useful for book authors as it is expected to be an example of the availability of analogies.


There have been no studies of the analogies in the elementary school (between the 5th and 8th grades) textbooks where mathematics is being taught as a subject in Turkey. From this point of view, it was aimed to investigate the analogies in the elementary school mathematics textbooks.

**Conceptual Framework**

**Definition of Analogies**

According to Glynn (2008), an analogy is a mapping based on similarities between a concept and another concept. According to Lawson (1993), analogies are the use of a concept that is well known to the learners in the mind of the learner for a new concept to learn. Palmquist (1996) defines analogism as another analogy well-known by learners with similar characteristics but not identical, which is used to understand a concept that is newly learned and generally abstract. According to a similar definition (Durmuş, 2013), the analogy between the well-known concept and the not-well-known concept in analogies, similarities and differences, facilitates the understanding of a concept that is not well known.

Palmquist (1996) stated that an analogy should have the following properties to be good:

- Analogies should be rich in structure.
- Analogies should be the diversity of the newly learned concept in terms of the analogous understanding,
- Analogies should not cause misconception in learners.
- Analogies must be applicable to the newly learned concept.
- Analogies should not be differentiated by all learners.
Gentner (1983) described the existence of two elements when describing analogies: source or analog concept (base) and target concept (target). Analogy is a function to achieve the target cognition using the familiar analogue (Gentner, 1983). While analog is a well-known concept by learners, the goal is a newly learned concept (Gilbert, 1997). According to Glynn, Taasoobshirazi and Fowler (2007), analogy can only be achieved when analog and target concepts share common characteristics. Glynn (2008) charted this situation as follows and gave an example between water cycle and electric cycle (See Figure 1).

![Figure1. A conceptual representation of an analogy and an example](image)

Glynn (2008) noted that analgesics should be used with caution when teaching:
- The target concept should be clearly indicated.
- The analog concept should be organized according to the target concept.
- The similarities that must be between the analog and the target should be clearly stated.
- Similarities between analogue and target concept should be compared.
- Situations in which the generated analogue does not work should also be mentioned.
- The results to be derived from the analogue should be established.

In addition to this, if the analogy is to be used for the first time teaching a concept, teaching the analogy should not be unfamiliar to the intended audience (Thiele and Treagust, 1994). Learning by using of an analogy that the learner has never seen before and that is not informed or contrary to the way of thinking can lead to misconceptions by learning (Aykutlu ve Şen, 2012). According to Iding (1997), in order to avoid this negative situation, it is necessary to clearly state the characteristics between analogue and target concepts.

**Types of Analogies**

Analogies can be classified as follows according to the relation between analogue and target concept (Demirci-Güler and Yağbasan, 2008):

1. Structural analogies: It is based on external similarities that are evident between analogue and target. For example, "The diaphragm is like a reversed tea plate." In this example, the diaphragm rim and the tea plate are just like the outer appearance.
2. Functional analogies: Analogue is based on the behavior or functions of the target. For example, "The springs for the spontaneous closure of the doors resemble the muscles attached to the bones." In this analogy, the springs and muscles are functionally similar.
3. Structural-functional analogies: Analogue and target must both have external similarities and have the same function.
Analogies can be classified as follows according to the presentation form of the relationship between the analog concept (Demirci-Güler and Yağbasan, 2008):

1. Verbal analogy: The analogy between analog and target is to be expressed in words only.
2. Pictorial analogy: This is the use of images other than words in the kind of analogy.
3. Verbal-pictorial analogy: In this kind of analogy, there must be both verbal expressions and pictures.

Classification can also be made according to the level of abstraction in relation to the analogy between analogies (Demirci-Güler and Yağbasan, 2008):

1. Abstract-concrete analogy: concrete concepts are used while abstract concepts are explained in this kind of analogy.
2. Abstract-abstract analogy: abstract concepts are used while abstract concepts are described in this kind of analogy.
3. Concrete-concrete analogy: concrete concepts are used when explaining the concrete concepts that are concrete in this kind of analogy.

According to the presentation of the analogy, classification can be made as follows (Demirci-Güler and Yağbasan, 2008):

1. Preliminary organizational analogies: The analogy to be taught in this type of analogy is used before analogy is given to the subject. In this way, learners are given a preliminary knowledge of the subject and their attention is drawn.
2. Activating analogies: It is aimed to reinforce the knowledge about the concept to be taught in this analogy type with examples.
3. Final organizational analogies: In this analogy type, an analogy is used to reconstruct the subject matter of the new concept to be taught.

Classification of analogies according to their wealth status is as follows (Demirci-Güler and Yağbasan, 2008):

1. Simple analogies: The relationship between analogy and analogy is not explicitly stated.
2. Enriched analogy: In contrast to the simple analogy, analogy between the analogue and the target is used to explain why analogy is used.
3. Extended analogy: In the case of this analogy, the established analogy is used more than once, spreading all over the subject.

Şeyihoğlu and Gürbüz (2015), which examines the analogy types above and departed from many studies, has made the analogy classification presented in the summary in Figure 2.
In this study, the above classification as summarized by Şeyihoğlu and Gürbüz (2015) is used.

**Methodology**

*Paradigm of research*

There are two types of approaches used in educational research: qualitative and quantitative approaches. While the bases of quantitative approaches are nourished by positivism, the origins of qualitative approaches are based on paradigm beyond positivism. The qualitative research approach is an approach that takes the concept of research into the environment in which the social phenomena are connected and takes the foreground to the meaning (Yıldırım and Şimşek, 2008). In recent years, social sciences and educational research have been carried out under the light of interpretative paradigms based on qualitative evidence (Kertil, 2008).

Qualitative research is an umbrella term encompassing a descriptive / interpreting sequence that aims to mean, not frequency / frequency, that is aimed at describing, interpreting, transforming a phenomenon in the social world in more or less natural environments (Merriam, 2009). Qualitative research differs from quantitative research in many ways (Yıldırım and Şimşek, 2008). In qualitative research it is assumed that variables are not meaningful on their own, that each variable is influenced by other related variables, and that this association leads to the real meaning of the relevant variable and it is tried to show the coexistence of these variables instead of examining the variables independently of each other (Yıldırım and Şimşek, 2008). Qualitative research provides more in-depth information on social events and psychological measures according to quantitative research and provides in-depth information on social events in relation to quantitative research methods (Büyüköztürk et al., 2009). This research has been carried out with a qualitative approach due to the topic and research question and for the reasons stated above, and a qualitative (interpreting) approach has been sought to answer research questions.

*Research Design*

Each research problem requires a unique research design and data analysis strategy since there are no pre-determined rules and standard approaches that can be applied to different environments and
groups in qualitative research (Yıldırım and Şimşek, 2008). The research of this research was conceived as an analysis of a document in terms of examining the phenomenon of ‘examining the analogies used in secondary school textbooks’.

Document analysis is defined as the analysis of written materials containing various information about the events or events for which the research is intended (Yıldırım and Şimşek, 2008). Content analysis of documents that have an important place in social trends, historical documents and cultural studies has begun to be used in parallel with the search for multiple methods emerging in educational researches (Çeken and Eş, 2013).

The documents examined by this research subject were approved by the Ministry of National Education and designated as secondary school (5th, 6th, 7th and 8th grade) mathematics textbooks used as textbooks in Turkey

**Findings**

A total of 29 analogies are recognized in the analyzed textbooks. Table 1 offers an overview of textbook analogies by summarizing the distribution through the different categories and analyzed textbooks.

<table>
<thead>
<tr>
<th>Type of Analogies</th>
<th>5th grade 1st semester (MoNE)</th>
<th>5th grade 2nd semester (MoNE)</th>
<th>5th grade (Private)</th>
<th>6th grade (MoNE)</th>
<th>6th grade (Private)</th>
<th>7th grade (Private)</th>
<th>8th grade (Private)</th>
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<tbody>
<tr>
<td>Structural</td>
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<td>Functional</td>
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<tr>
<td>Structural-Functional</td>
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<td>1</td>
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<tr>
<td>Presentation format</td>
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<td></td>
<td>Pictoral</td>
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<td>2</td>
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<td>4</td>
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<td></td>
<td>Verbal-Pictoral</td>
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<td>3</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>24</td>
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<td>Abstraction</td>
<td>Concrete-Concrete</td>
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<td>Concrete-Abstract</td>
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<td></td>
<td>Concrete-Abstract</td>
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<td>5</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
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<td></td>
<td>Position of Analog Relative to Target</td>
<td>Preliminary</td>
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<td>2</td>
<td>6</td>
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<td>3</td>
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<td></td>
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<td>Level of wealth</td>
<td>Simple</td>
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<td>4</td>
<td>6</td>
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<td>3</td>
<td>5</td>
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<td>Personal Analogies</td>
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Table 1. Overview of Textbook Analogies
A total of 7 analogies were identified in the first and second semester of the 5th grade textbook of the Ministry of National Education, and a total of 6 analogies were found in the 5th grade private publishing course textbook. In the 6th grade textbook published by Ministry of National Education, 3 pieces and in the 6th grade private publishing course textbook, 3 pieces of analogy were determined. Since there is no 7th grade and 8th grade textbooks of Ministry of Education publication, only 5th and 5th analogies were found in the 7th grade private publishing course book and the 8th grade private publishing course text book. Examples of analogies are as follows.

**Figure 3.** An analogy in MoNE – 5th grade 1st semester textbook

According to this analogy, the traces left on the sea are analogous to parallel and cross lines. This analogy is classified as:

- Structural: The source and the target physical appearance are similar in terms of formation.
- Verbal-pictoral: The similarity between the source and the target is presented verbally and is supported by a picture.
- Abstract-concrete: The source is concrete and the target is abstract.
- Activator: It is used in the subject during teaching.
- Simple: There exists no reason or limitation between the source and the target.

**Figure 4.** An analogy in MoNE – 6th grade textbook

According to this analogy, the scallop and the minute hand are likened to the concept of angle. This analogy is classified as:

- Structural: The source and the target physical appearance are similar in terms of formation.
- Pictoral: The similarity between the source and the target is presented only pictorally.
- Abstract-concrete: The source is concrete and the target is abstract.
- Preliminart: It is used in the beginning of subject teaching.
- Simple: There exists no reason or limitation between the source and the target.
Conclusion and Discussion

In this study aimed to determine the analogies in the 5th, 6th, 7th and 8th grade mathematics textbooks of the secondary school, 29 analogies were determined as the result of the findings. A total of 7 analogies were identified in the first and second semester of the 5th grade textbook of the Ministry of National Education, and a total of 6 analogies were found in the 5th grade private publishing course textbook. In the 6th grade textbook published by Ministry of National Education, 3 pieces and in the 6th grade private publishing course textbook, 3 pieces of analogy were determined. Since there is no 7th grade and 8th grade textbooks of Ministry of Education publication, only 5th and 5th analogies were found in the 7th grade private publishing course book and the 8th grade private publishing course text book. According to this, 7.25 analogies per textbook are analyzed.

When we look at the number of analogies for textbooks made in the field, it is observed that 71 analogies were found in the geography textbooks of 9th, 10th, and 12th grades examined by Şeyihoğlu and Özgürbüz (2015). Azizoğlu, Çamurcu and Kirtak (2014) examined a total of 46 analogies in the 9th, 10th, 11th and 12th class physics textbooks. It is seen that the average is 11.5. In the chemistry textbooks analyzed by Azizoğlu, Aslan & Pekcan (2013) 39 analogies were found and it was seen that the average was 9.75. A total of 89 analogies were determined in the books of the 4th, 5th, 6th, 7th and 8th grade primary science and technology lessons examined by Demirci-Güler and Yağbasan (2008). It is seen that the average is 17.8. Accordingly, it can be concluded that the number of analogies obtained from this research is less than other studies. It can be assumed that this situation is due to the nature of the course being studied; Because the studies listed above are generally regarded as science courses.

Similar to this result, in the study done by Ünver (2009) within the scope of mathematics lesson, he found 20 analogies at the level of the 9th grade mathematics course book. This number is also considerably higher than the number of analogies in each of the secondary school mathematics textbooks examined in this study. Thus, numerically and on an average basis, the analgesia studied in this study can be achieved as a result.

One of the sub problems of this research concerns the comparison of textbooks published by the Ministry of National Education and private publishing houses in terms of analogy use. In this respect, only a comparison can be made at the 5th and 6th grade levels. The number of analogies in the 5th grade middle school mathematics textbooks was found to be 7 in the 1st and 2nd semesters in the MEB publication and 6 in the 5th class private publishing course textbook. According to this, at the level of this class can be reached the result that there is 1 more analogy in the book published by MONE.

The number of analogies included in the 6th grade middle school mathematics textbooks was found to be 3 in the MEB publication and 3 in the 5th grade private publishing course textbook. According to this, at the level of this class, the book published by the Ministry of Education and the book published by the private publisher can be reached as the result of having an equal number of analogies. Considering that the number of analogies in this study is less than the field text, it can be considered that both MONE and private publishing houses are not sufficiently used for the analogy in textbooks.

Of the 29 analogies detected in this study, 26 were found to be structurally related only to the relation. This result does not coincide with field studies. For example, none of the 20 analogies that Unver (2009) found in the study are not only structural, only 2 are structural-functional. Similarly, in studies of other branches of science, structural-functional biochemistry books (Orgill and Bodner, 2006) are more functional than in chemistry books (Thiele and Treagust, 1994) and biology books (Thiele, Venville and Treagust, 1995) Analogy was used.
Twenty-four of the 29 analogies detected in this study were verbal-pictured according to the presentation format. In terms of the presentation of the analogy, it is stated that in mathematics textbooks, analogies are usually verbal-pictorial, the limitations are not defined and teacher-centered (Thiele and Treagust, 1994). At the same time, it has been suggested that verbal-pictorial presentation is more effective in learning concepts (Bean, Searles and Cowen, 1990). Thiele and Treagust (1994) reported that they had more analogy in terms of atomic structure, ties and energy, such as difficult and abstract concepts such as chemistry textbooks, and that these analogies were supported visually. However, 28 of the 29 analogies identified in this study were simply determined by the level of wealth. These findings are also consistent with the findings of other studies on the subject. For example, Kobak (2013) found that 29 of the 39 analogies identified in secondary school chemistry textbooks are simple. In the study conducted by Demirici-Güler and Yaşabasancan (2008), 82% of the analogies detected are simple analogies. On the other hand, the result obtained from this research does not coincide with the work of Unver (2009). Only three of the 20 mathematical analogies found in Unver’s (2009) study are simple. This can be due to class level or subject-related changes; Because in this study middle school textbooks are examined while Unver’s (2009) study is the 9th grade textbook of secondary school.

Suggestions

The following suggestions can be made at the end of the research and other researches:

1. Analogies are used in the textbooks, but instead of simple analogies, enriched or extended analogies should be used more.
2. Explanation of misconceptions that may occur in the use of analogies
3. Analogue and non-target aspects should be emphasized so that textbooks can be prevented from perceiving the same analogy in the curriculum.
4. The analogies in the textbooks can be supported with pictures to enable the students to grasp more easily.
5. It is useful to specify the locations where the analysts go to the field.
6. Analogies should be designed in a motivational way to new subject by also grasping the students’ prior knowledge about the topic.
7. Analogies should be appropriate to the level of students’ knowledge. Analogies that do not fit the cognitive level of the learners may cause misconceptions of the students as well as misinterpretation between the analog and the target concepts.

References


